Digital Linear Filter (DLF) design

Some notes regarding the `fdesign` add-on for `empymod`

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1 About and Info

The add-on `fdesign` can be used to design digital linear filters for the Hankel or Fourier transform, or for any linear transform. For this included or provided theoretical transform pairs can be used. Alternatively, one can use the EM modeller `empymod` (Werthmüller, 2017) to use the responses to an arbitrary 1D model as numerical transform pair.

More information can be found in the following places:

- The article about `fdesign` is in the repo github.com/empymod/article-fdesign
- Example notebooks to design a filter can be found in the repo github.com/empymod/empymod-examples

The methodology of `fdesign` is based upon Kong (2007). The whole project of `fdesign` started with the Matlab scripts from Kerry Key, which he used to design his filters for Key (2009, 2012). Fruitful discussions with Evert Slob and Kerry Key improved the add-on substantially.

Note that the use of `empymod` to create numerical transform pairs is, as of now, only implemented for the Hankel transform (via `fdesign.empy_hankel`).

2 Implemented analytical transform pairs

The following tables list the transform pairs which are implemented by default. Any other transform pair can be provided as input. A transform pair is defined in the following way:

```
from empymod.scripts.fdesign import Ghosh

def my_tp_pair(var):
    """My transform pair."""

    def lhs(l):
        return func(l, var)

    def rhs(r):
        return func(r, var)

    return Ghosh(name, lhs, rhs)
```

Here, `name` must be one of `j0`, `j1`, `sin`, or `cos`, depending what type of transform pair it is. Additional variables are provided with `var`. The evaluation points of the `lhs` are denoted by `l`, and the evaluation points of the `rhs` are denoted as `r`. As an example here the implemented transform pair `j0_1`:

```
def j0_1(a=1):
    """Hankel transform pair J0_1 ([Anderson_1975]_)."""

    def lhs(l):
        return l*np.exp(-a*l**2)
```

```
def rhs(r):
    return np.exp(-r**2/(4*a))/(2*a)

return Ghosh('j0', lhs, rhs)
```

2.1 Implemented Hankel transforms

Name	Reference	Transform pair
j0_1	Anderson (1975)	$$\int_0^\infty l \exp\left(-al^2\right) J_0(lr)\,\mathrm{d}l = \frac{\exp\left(\frac{-r^2}{4a}\right)}{2a} \quad (1)$$
j0_2	Anderson (1975)	$$\int_0^\infty \exp\left(-al\right) J_0(lr)\,\mathrm{d}l = \frac{1}{\sqrt{a^2+r^2}} \quad (2)$$
j0_3	Guptasarma and Singh (1997)	$$\int_0^\infty l \exp\left(-al\right) J_0(lr)\,\mathrm{d}l = \frac{a}{(a^2+r^2)^{3/2}} \quad (3)$$
j0_4	Chave and Cox (1982)	$$\int_0^\infty \frac{l}{\beta} \exp\left(-\beta z_\mathrm{v}\right) J_0(lr)\,\mathrm{d}l = \frac{\exp\left(-\gamma R\right)}{R} \quad (4)$$
j0_5	Chave and Cox (1982)	$$\int_0^\infty l \exp\left(-\beta z_\mathrm{v}\right) J_0(lr)\,\mathrm{d}l = \frac{z_\mathrm{v}(\gamma R+1)}{R^3} \exp\left(-\gamma R\right) \quad (5)$$
j1_1	Anderson (1975)	$$\int_0^\infty l^2 \exp\left(-al^2\right) J_1(lr)\,\mathrm{d}l = \frac{r}{4a^2} \exp\left(-\frac{r^2}{4a}\right) \quad (6)$$
j1_2	Anderson (1975)	$$\int_0^\infty \exp\left(-al\right) J_1(lr)\,\mathrm{d}l = \frac{\sqrt{a^2+r^2}-a}{r\sqrt{a^2+r^2}} \quad (7)$$
j1_3	Anderson (1975)	$$\int_0^\infty l \exp\left(-al\right) J_1(lr)\,\mathrm{d}l = \frac{r}{(a^2+r^2)^{3/2}} \quad (8)$$
j1_4	Chave and Cox (1982)	$$\int_0^\infty \frac{l^2}{\beta} \exp\left(-\beta z_\mathrm{v}\right) J_1(lr)\,\mathrm{d}l = \frac{r(\gamma R+1)}{R^3} \exp\left(-\gamma R\right) \quad (9)$$
j1_5	Chave and Cox (1982)	$$\int_0^\infty l^2 \exp\left(-\beta z_\mathrm{v}\right) J_1(lr)\,\mathrm{d}l = \frac{r z_\mathrm{v}(\gamma^2 R^2+3\gamma R+3)}{R^5} \exp\left(-\gamma R\right) \quad (10)$$

$$a > 0, \quad r > 0 \quad (11)$$
$$z_\mathrm{v} = |z_\mathrm{rec} - z_\mathrm{src}| \quad (12)$$
$$R = \sqrt{r^2 + z_\mathrm{v}^2} \quad (13)$$
$$\gamma = \sqrt{2j\pi\mu_0 f/\rho} \quad (14)$$
$$\beta = \sqrt{l^2+\gamma^2} \quad (15)$$
$$(16)$$

2.2 Implemented Fourier transforms

Name	Reference	Transform pair	
sin_1	Anderson (1975)	$\int_0^\infty l\exp\left(-a^2l^2\right)\sin(lr)\,\mathrm{d}l = \frac{\sqrt{\pi}r}{4a^3}\exp\left(-\frac{r^2}{4a^2}\right)$	(17)
sin_2	Anderson (1975)	$\int_0^\infty \exp\left(-al\right)\sin(lr)\,\mathrm{d}l = \frac{r}{a^2+r^2}$	(18)
sin_3	Anderson (1975)	$\int_0^\infty \frac{l}{a^2+l^2}\sin(lr)\,\mathrm{d}l = \frac{\pi}{2}\exp\left(-ar\right)$	(19)
cos_1	Anderson (1975)	$\int_0^\infty \exp\left(-a^2l^2\right)\cos(lr)\,\mathrm{d}l = \frac{\sqrt{\pi}}{2a}\exp\left(-\frac{r^2}{4a^2}\right)$	(20)
cos_2	Anderson (1975)	$\int_0^\infty \exp\left(-al\right)\cos(lr)\,\mathrm{d}l = \frac{a}{a^2+r^2}$	(21)
cos_3	Anderson (1975)	$\int_0^\infty \frac{1}{a^2+l^2}\cos(lr)\,\mathrm{d}l = \frac{\pi}{2a}\exp\left(-ar\right)$	(22)

References

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