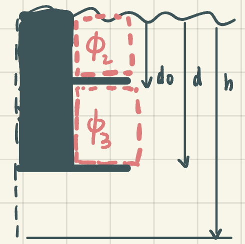


Koh, H. J., & Cho, I. H. (2016). Heave motion response of a circular cylinder with the dual damping plates. *Ocean Engineering*, *125*, 95-102.

ϕ_2

In region (II), the radiation potential satisfies the Laplace equation, free surface, and body boundary conditions at upper damping plate ($\partial\phi^{(II)}/\partial z = 1$, at $z = -d_0$) and side wall of the cylinder ($\partial\phi^{(II)}/\partial r = 0$, at $r = b$). The velocity potential in regions (II) can be written as the sum of a particular solution and a homogeneous solution.

$$\varphi^{(II)}(r,z) = \psi_P^{(II)}(r,z) + \sum_{n=0}^{\infty} B_n\left(I_0(k_{2n}r) - \frac{I_0'(k_{2n}b)}{K_0'(k_{2n}b)}K_0(k_{2n}r)\right)f_{2n}(z), \tag{5}$$

where I_0 is the modified Bessel function of the first kind. The prime appearing in the superscript denotes the derivative with respect to the argument. The eigenvalues $(k_{20} = -ik_2,\ k_{2n},\ n = 1, 2, \cdots)$ in region (II) are the roots of the dispersion relation $(k_{2n} \tan k_{2n}d_0 = -\omega^2/g)$, and the normalized vertical eigenfunctions $f_{2n}(z)$ are defined as follows:

$$f_{2n}(z) = N_{2n}^{-1} \cos k_{2n}(z + d_0), \quad n = 0, 1, 2, \ldots$$
$$(N_{2n})^2 = \frac{1}{2}\left(1 + \frac{\sin 2k_{2n}d_0}{2k_{2n}d_0}\right). \tag{6}$$

The eigenfunctions $f_{2n}(z)$ satisfy the orthogonal relation.

$$\frac{1}{d_0}\int_{-d_0}^{0} f_{2m}(z)f_{2n}(z)dz = \delta_{mn}. \tag{7}$$

The particular solutions in region (II) satisfying the inhomogeneous body boundary condition can be written as follows:

$$\psi_P^{(II)}(r,z) = z + \frac{1}{K}, \tag{8}$$

$K = \frac{w^2}{g}$

ϕ_3

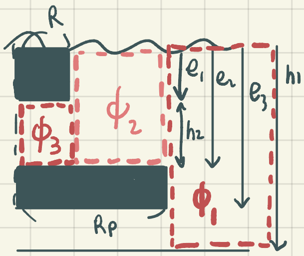
In region (III), the velocity potential that satisfies the body boundary conditions at the damping plates ($z = -d_0, -d$) and side wall can be written as the sum of a particular and a homogeneous solution.

$$\varphi^{(III)}(r,z) = \psi_P^{(III)}(r,z) + \sum_{n=0}^{\infty} \varepsilon_n C_n\left[I_0(\alpha_n r) - \frac{I_0'(\alpha_n b)}{K_0'(\alpha_n b)}K_0(\alpha_n r)\right]\cos \alpha_n(z + d), \tag{9}$$

where ε_n is the Neumann symbol, defined by $\varepsilon_n = 1$, if $n = 0$, and $\varepsilon_n = 2$, if $n \geq 1$, and the eigenvalues in region (III) is $\alpha_n = n\pi/c_0$, $(n = 0, 1, \cdots)$. The particular solutions in region (III) can be written by:

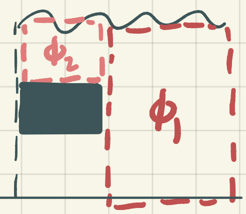
$$\psi_P^{(III)}(r,z) = \frac{1}{2c_0}\left[(z + d)^2 - (z + d_0)^2\right]. \tag{10}$$

$c_0 = d - d_0$



Hassan, M., Bora, S. N., & Biswakarma, M. (2020). Water wave interaction with a pair of floating and submerged coaxial cylinders in uniform water depth. *Marine Systems & Ocean Technology*, *15*, 188-198.

Borah, P., & Hassan, M. (2021). Scattering of water waves by a wave energy device consisting of a pair of co-axial cylinders in a uniform water having finite channel width. *Journal of Ocean Engineering and Science*, *6*(3), 276-284.



Jiang, S. C., Gou, Y., & Teng, B. (2014). Water wave radiation problem by a submerged cylinder. *Journal of Engineering Mechanics*, *140*(5), 06014003.

ϕ_1

In domain Ω_1, the potential can be written in terms of the following well-known eigenfunction expansion (Ursell 1976; Wu et al. 2004)

$$\phi_1(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \cos m\theta A_{ml} P_m(k_l r) Z_l(k_l z) \tag{13}$$

where A_{ml} $(l, m = 0, 1, 2, \ldots)$ = unknown coefficients. The dispersion relation is

$$\omega^2 = gk \tanh kd$$

The positive real root defined by k_0 is the wave number of the progressive mode, and the imaginary roots, ik_l for $l \geq 1$, are the wave numbers of the evanescent modes, which are only of local importance. The radial eigenfunctions $P_m(k_l r)$ are given by

$$P_m(k_l r) = \begin{cases} H_m(k_l r)/H_m(k_l a) & l = 0 \\ K_m(k_l r)/K_m(k_l a) & l \geq 1 \end{cases}$$

where H_m = the first kind of Hankel function of order m, which satisfies the radiation condition in Eq. (9); and K_m is the second kind of modified Bessel function of order m. The vertical eigenfunctions $Z_l(k_l z)$ form an orthogonal set in $[-d, 0]$ and are defined as

$$Z_l(k_l z) = \begin{cases} \cosh k_0(z + d)/\cosh k_0 d & l = 0 \\ \cos k_l(z + d)/\cos k_l d & l \geq 1 \end{cases}$$

The inner products of these functions are

$$N_{Zl} = \int_{-d}^{0} Z_l^2(k_l z)dz = \begin{cases} \frac{1}{\cosh^2 k_0 d}\left(\frac{d}{2} + \frac{\sinh 2k_0 d}{4k_0}\right) & l = 0 \\ \frac{1}{\cos^2 k_l d}\left(\frac{d}{2} + \frac{\sin 2k_l d}{4k_l}\right) & l \geq 1 \end{cases}$$

ϕ_2

In domain Ω_2, the free-surface condition in Eq. (4) and the body-surface condition in Eq. (7) at the upper side of the cylinder need to be satisfied. After some derivation, the authors obtain the following results:

$$\phi_2(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \cos m\theta B_{mj} Q_m(\lambda_j r) U_j(\lambda_j z) + \phi_2^p(r,\theta,z) \tag{14}$$

where B_{mj} $(m, j = 0, 1, 2, \ldots)$ = unknown coefficients; and λ_0 and $i\lambda_j$ are the real and the imaginary eigenvalues of the dispersion equation

$$\omega^2 = g\lambda \tanh \lambda d_1$$

in domain Ω_2. The radial function $Q_m(\lambda_j r)$ is defined as follows:

$$Q_m(\lambda_j r) = \begin{cases} J_m(\lambda_j r)/J_m(\lambda_j a) & j = 0 \\ I_m(\lambda_j r)/I_m(\lambda_j a) & j \geq 1 \end{cases}$$

where I_m = first kind of modified Bessel function of order m. Similar to the case of domain Ω_1, the vertical eigenfunctions $U_j(\lambda_j z)$ are written as

$$U_j(\lambda_j z) = \begin{cases} \cosh \lambda_0(z + d_1)/\cosh \lambda_0 d_1 & j = 0 \\ \cos \lambda_j(z + d_1)/\cos \lambda_j d_1 & j \geq 1 \end{cases}$$

The inner products of these functions are

$$N_{Yj} = \int_{-d_1}^{0} U_j^2(\lambda_j z)dz = \begin{cases} \frac{1}{\cosh^2 \lambda_0 d_1}\left(\frac{d_1}{2} + \frac{\sinh 2\lambda_0 d_1}{4\lambda_0}\right) & j = 0 \\ \frac{1}{\cos^2 \lambda_j d_1}\left(\frac{d_1}{2} + \frac{\sin 2\lambda_j d_1}{4\lambda_j}\right) & j \geq 1 \end{cases}$$

In Eq. (14), $\phi_2^p(r,\theta,z)$ = the particular solution satisfying the body condition in Eq. (7) and the free surface condition from Eq. (4). Its expression is dependent on the form of $v_2(\theta)$.

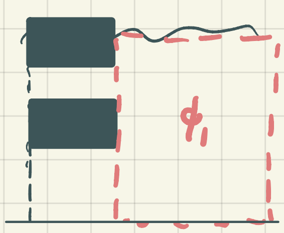
Surge: $\phi_2^p = 0$

Heave

$$\phi_2^p = \sum_{n=1}^{\infty} \frac{2J_0(q_n r)}{q_n^2 a J_1(x_{0n})} \frac{gq_n \cosh q_n z + \omega^2 \sinh q_n z}{\omega^2 \cosh q_n d_1 - gq_n \sinh q_n d_1} \quad \text{in } \Omega_2 \tag{32}$$

Pitch

$$\phi_2^p = -\cos\theta\left[\sum_{n=1}^{\infty} \frac{2J_1(h_n r)}{h_n^2 J_2(x_{1n})} \frac{gh_n \cosh h_n z + \omega^2 \sinh h_n z}{\omega^2 \cosh h_n d_1 - gh_n \sinh h_n d_1}\right]$$
in Ω_2



Berggren, L., & Johansson, M. (1992). Hydrodynamic coefficients of a wave energy device consisting of a buoy and a submerged plate. *Applied Ocean Research*, *14*(1), 51-58.

ϕ_1

The formulation starts from the potentials developed independently in each subdomain. Applying the method of separation of variables gives the spatial potentials in ach region expressed in terms of orthogonal series. In region I the potential becomes

$$\phi_1 = \sum_{n=1}^{\infty} A_n \cos \lambda_n(z + h_1) \frac{R_n(\lambda_n r)}{R_n(\lambda_n R)} \tag{13}$$

where the eigenvalues are given by

$$\begin{aligned} \lambda_1 &= -ik \quad \text{where } k \text{ is the wavenumber} \\ k \tanh kh_1 &= \omega^2/g \qquad n = 1 \\ \lambda_n \tanh \lambda_n h_1 &= -\omega^2/g \qquad n = 2, 3 \ldots \end{aligned} \tag{14}$$

and the radial function R_n is given by

$$\begin{aligned} R_1(\lambda_1 r) &= H_0^{(1)}(i\lambda_1 r) = H_0^{(1)}(kr) \qquad n = 1 \\ R_n(\lambda_n r) &= K(\lambda_n r) \qquad n = 2, 3 \ \ldots \end{aligned} \tag{15}$$

where $H_0^{(1)}$ is the Hankel function of first kind and zeroth order and K_0 is the modified Bessel function of second kind and zeroth order.