Removing degeneracy and multimodality from gravitational wave parameters

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Abstract

Binary mergers are described by 15 parameters, of which only ~ 10 independent combinations are typically constrained to varying degree.
We define coordinates that disentangle well- and poorly-measured quantities, removing correlations. Additionally, we identify approximate discrete symmetries in the likelihood as the primary cause of multimodality in the posterior, and design a method to remove this type of multimodality.
The resulting posteriors have little structure and can be sampled efficiently and robustly with the open-source software `cogwheel` [1].

Observables

Extrinsic parameters mainly affect the amplitude, phase and time of the waveform at each detector [2]:

$$h_k \approx a_k e^{i\varphi_k} e^{-i2\pi f t_k} h_0(f;\theta_{\rm int}); \qquad k \in \text{detectors}$$

$$\begin{cases} a_k = \dfrac{\mathcal{M}^{5/6}}{d_L} |R_k(\iota, \alpha, \delta, \psi)| \\ \varphi_k = \arg R_k(\iota, \alpha, \delta, \psi) + 2\phi_{\rm ref} - 2\pi \bar{f} t_k \\ t_k = t_\oplus - \hat{\boldsymbol{N}}(\alpha, \delta) \cdot \boldsymbol{r}_k / c \end{cases}$$

These are measured to a precision given by the SNR,

$$\frac{\Delta a_k}{a_k} \sim \Delta\varphi_k \sim 2\pi\sigma_f \,\Delta t_k \sim \frac{1}{\rho_k}.$$

Notation

$$R_k = \frac{1+\cos^2\iota}{2} F_k^+(\alpha,\delta,\psi) - i\cos\iota\, F_k^\times(\alpha,\delta,\psi)$$

$$\sigma_f = \sqrt{\overline{f^2} - \overline{f}^2}, \text{ with } \overline{f^n} = \frac{\int df |h_0|^2 f^n / S(f)}{\int df |h_0|^2 / S(f)}$$

α, δ : sky location (equatorial system)
ι, θ_{JN} : instantaneous/average inclination
ψ : polarization
$\phi_{\rm ref}, t_\oplus$: coalescense phase and time

Removing degeneracy

We want coordinates that track a_k, t_k, φ_k separately. Let us sort detectors by SNR, $\rho_{k_0} > \rho_{k_1} > \dots$

- Trade distance for amplitude at reference detector:
$$d_L \to \hat{d} \equiv 1/a_{k_0}(d_L, \mathcal{M}, \iota, \alpha, \delta, \psi)$$
- Specify arrival time at reference detector
$$t_\oplus \to t_{k_0}(t_\oplus, \alpha, \delta)$$
- Parametrize sky location using "network" angles, where the z axis contains detectors k_0 and k_1:
$$\alpha, \delta \to \theta_{\rm net}, \phi_{\rm net}: \quad \cos\theta_{\rm net} \equiv \hat{\boldsymbol{N}}(\alpha,\delta) \cdot \hat{\boldsymbol{z}} = \frac{\Delta t_{01}}{\Delta t_{\rm max}}$$
- Trade orbital phase for phase at reference detector:
$$\phi_{\rm ref} \to \hat{\phi}_{\rm ref} \equiv \phi_{\rm ref} - \phi_{\rm ref}^{\rm pred}(\iota, \alpha, \delta, \psi, t_{k_0}) = \frac{\varphi_{k_0} - \varphi_{k_0}^{\rm best}}{2}$$
- Describe aligned spins using $\chi_{\rm eff}$.
- Set reference frequency in band [3], e.g. $f_{\rm ref} = \bar{f}$.

Removing multimodality

Approximate symmetries

There are 4 approximate discrete symmetries that often cause $2^4 = 16$-multimodality:

1. $\hat{\phi}_{\rm ref} \to \hat{\phi}_{\rm ref} + \pi$
2. $\psi \to \psi + \pi/2$
3. $\cos\theta_{JN} \to -\cos\theta_{JN}$
4. $\hat{\phi}_{\rm net} \to -\hat{\phi}_{\rm net}$

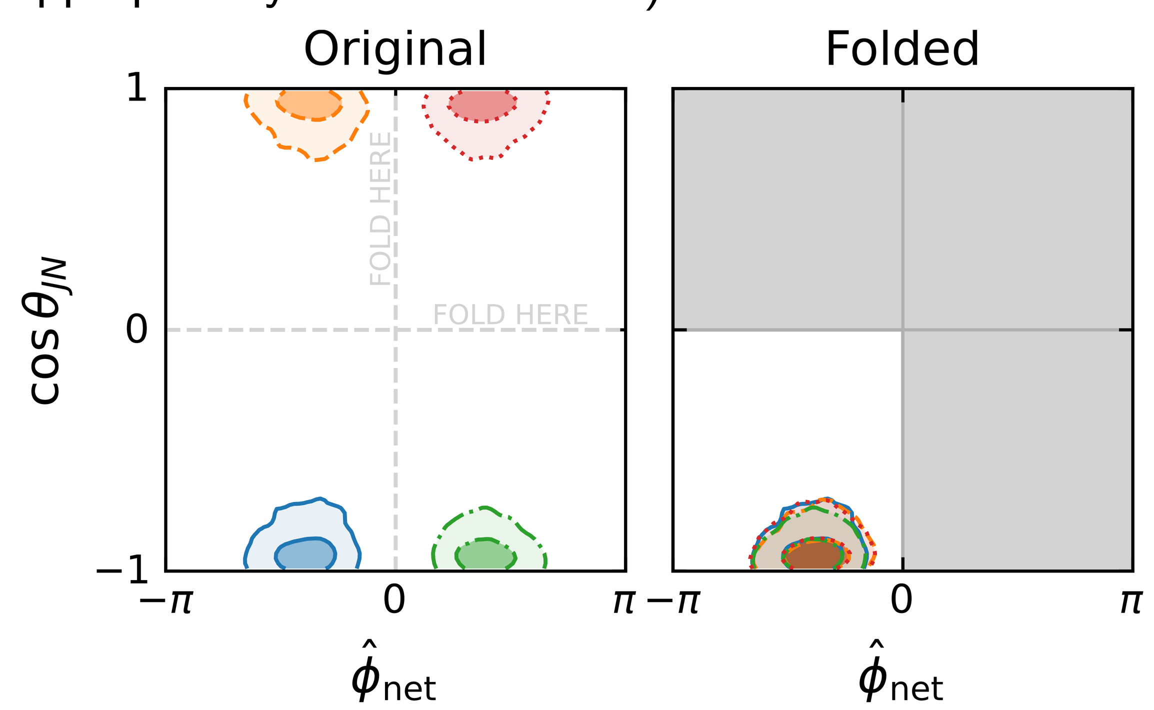
(where $\hat{\phi}_{\rm net} \equiv \phi_{\rm net} + \pi\,\Theta(\cos\theta_{JN})$; the spin azimuths require a similar redefinition).

1 and 2 are exact for quadrupolar waveforms, partially broken by higher modes. 3 and 4 hold best for multiple coaligned detectors ($\approx$ true for LIGO).

In our coordinates, these symmetries become one-parameter reflections or translations.

Folding

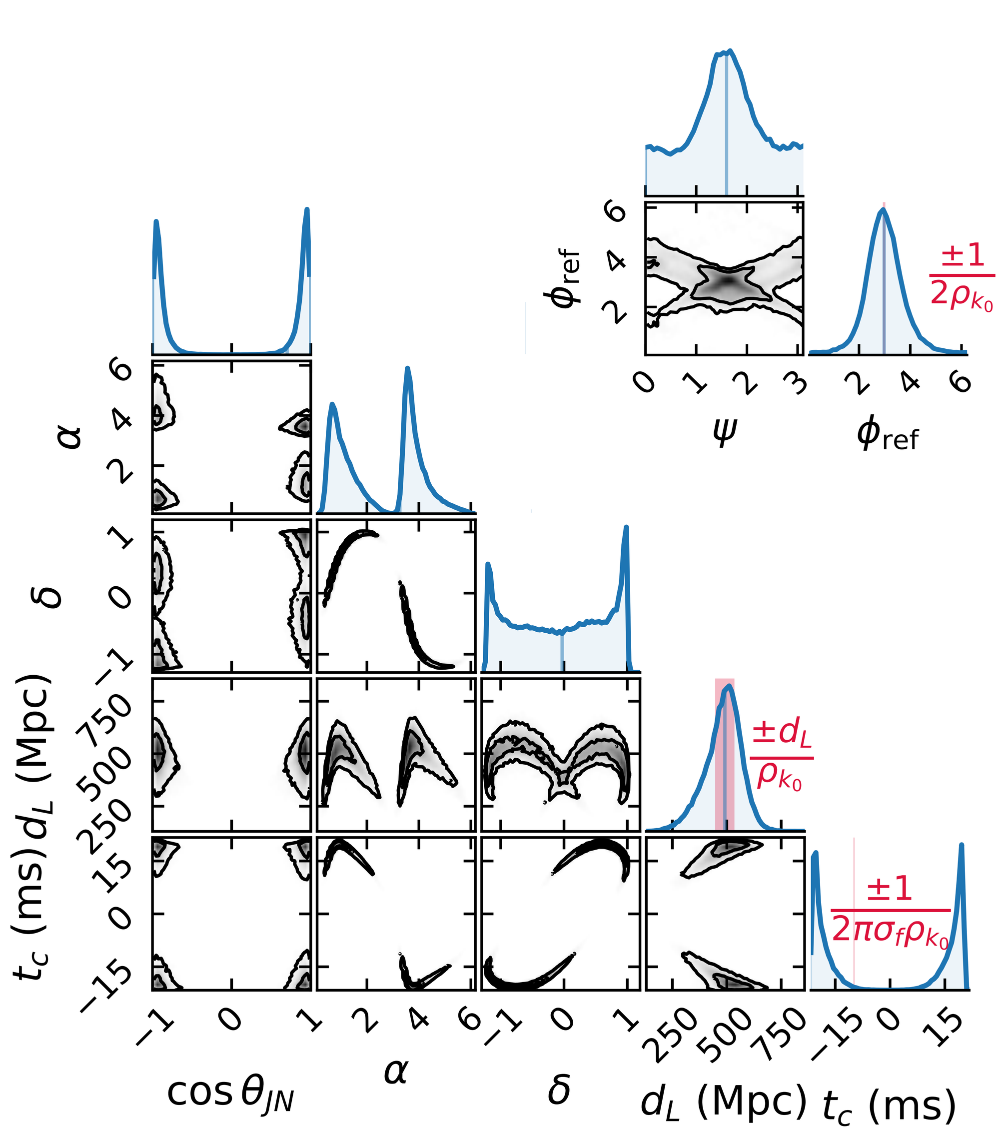
When multimodality arises from known approximate symmetries, we can "fold" the distribution (sum its appropriately reflected modes) to make it unimodal:



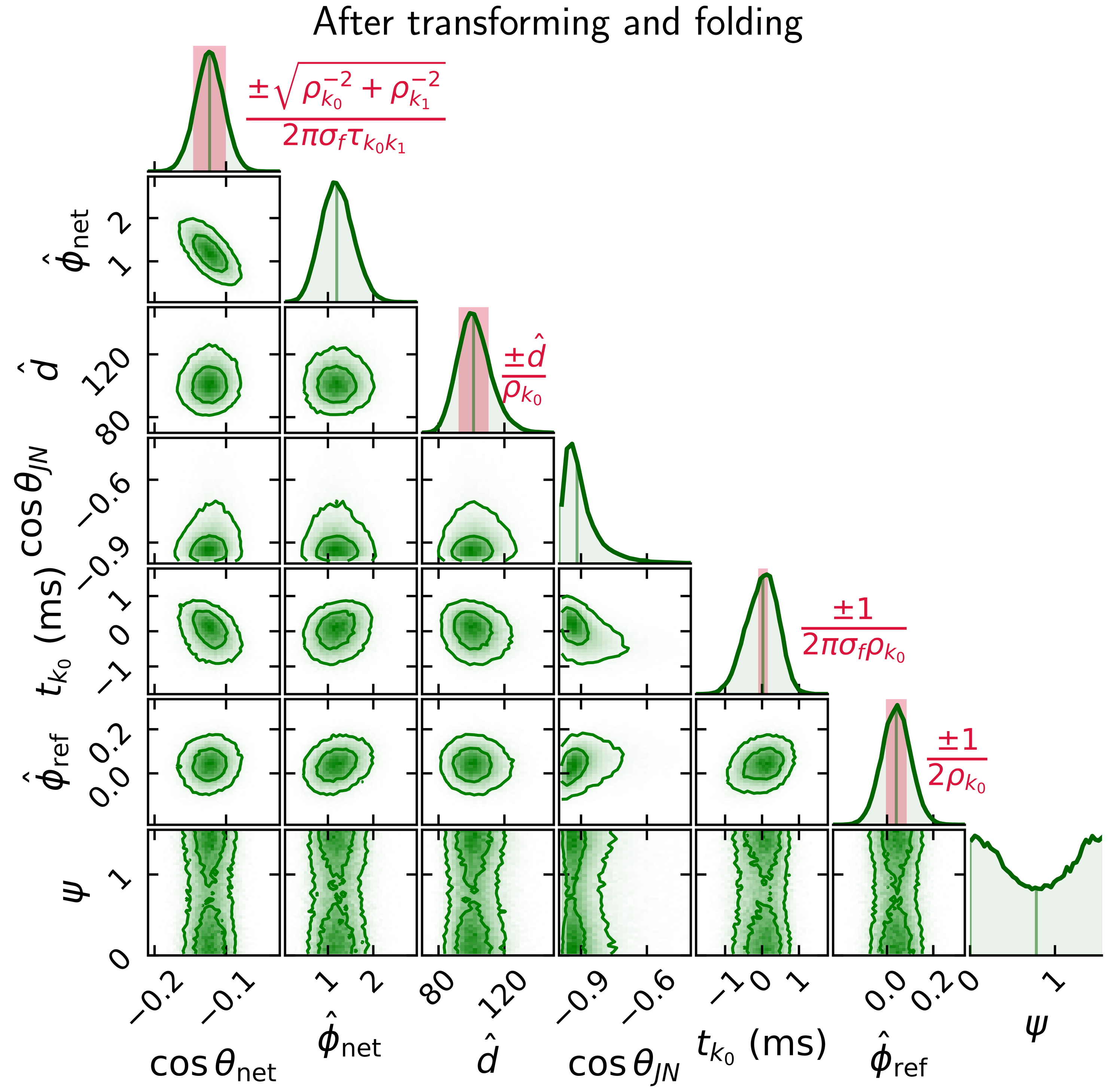
We sample the folded distribution and reconstruct the original in postprocessing.

Results

Before



After transforming and folding



Takeaway points

Our coordinates are ideal for sampling:

- Remove degeneracies
- Reduce multimodality by factor up to 16
- Simple Jacobian, invertible analytically

Code: github.com/jroulet/cogwheel.

References

[1] J. Roulet *et al.* (2022) PRD 106, 123015
[2] C. Cutler and É. E. Flanagan (1994) PRD 49, 2658
[3] B. Farr *et al.* (2014) PRD 90, 024018

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