# Agency MBS Valuation Model

## 1. Model Derivation

### Excess Returns

Total returns of fixed income assets have two primary components: income returns and price returns.

For spread asset classes like credit, it is often useful think of returns in excess of interest rates. This model estimates Carry as excess income returns and Value as excess price returns.

### MBS Valuation

With MBS, even returns in excess of interest rates are not independet of interest rates because MBS exhibit negative convexity due to the prepayment option. As interest rates decline, the price appreciation of MBS is limited by the increasing likelihood of prepayments. As interest rates rise, the price depreciation is exacerbated by the decreasing likelihood of prepayments. This negative convexity makes MBS valuation more complex. Credit spreads are more linear in their response to changes in interest rates and credit risk.

An MBS valuation model should thus consider spread reversion, prepayment optionality and their confluence. Provided ZV and OAS spreads are available, we can think of the differential as the embedded optionality or Convexity.

* **Zero Volatility Spread (ZV)**: Spread over interest rates, inclusive of optionality.
* **Option-Adjusted Spread (OAS)**: Spread after removing the impact of embedded options, particularly prepayments.
* **Convexity**: Defined as the differential between the Z-Spread and OAS, capturing the optionality premium intrinsic to MBS.

However, changes in OAS cannot be thought of as independent from Convexity. The dynamics of both OAS and Convexity are also likely influenced by interest rates.

I propose a **Joint Reversion Model** for OAS and Convexity, explicitly incorporating external factors such as interest rate volatility () and volatility of volatility (), while also accounting for model uncertainty inherent in OAS estimation.

### Model Components

#### 1. Joint Reversion Process for OAS and Convexity

* OAS follows a mean-reverting process toward a long-run equilibrium , integrating convexity and external influences:
* Convexity follows a mean-reverting process toward the convexity of Current Coupon (CC) bonds , which itself is influenced by the level of interest rates (not modelled here):
* and are parameters which describe the speed of reversion of the OAS and Convexity processes to their respective equilibriums.
* and are Wiener processes that introduce stochasticity into the OAS and Convexity processes. Scaled to the respective standard deviations of both processes, and capture those random fluctuations.

#### 2. Incorporation of Interest Rate Volatility and Volatility of Volatility

* **Rates Vol ()**: Influences both OAS and Convexity by altering prepayment risk and uncertainty.
* **Rates Vol of Vol ()**: Introduces regime-dependent fluctuations, affecting both the risk premium and convexity.

#### 3. Accounting for OAS Model Uncertainty

* OAS estimation depends on prepayment models, which become increasingly unreliable under heightened convexity conditions.
* We can introduce a convexity-dependent residual variance, ensuring greater dispersion in OAS innovations when convexity is elevated.

### Discretisation of the System of SDEs

To discretize the system of differential equations in the Joint Reversion Model for OAS and Convexity, we can use the Euler-Maruyama method, which is a straightforward approach for numerical integration of stochastic differential equations (SDEs). Here’s a step-by-step explanation of the process:

#### Continuous-Time Model

The continuous-time model for OAS and Convexity is given by:

1. **OAS Process**:

* where

1. **Convexity Process**:

#### Discretisation Using Euler-Maruyama Method

To discretise these equations, we approximate the continuous-time processes with discrete-time steps. Let be the time step size, and let for . The discretized versions of the SDEs are:

1. **Discretized OAS Process**:

* where

1. **Discretized Convexity Process**:

Here, and are independent standard normal random variables (i.e., $Z\_O, Z\_C (0, 1) $) representing the Wiener processes and .

## 2. Model Implementation

1. **Data Acquisition and Preprocessing**:
   * Collect historical time series for OAS, Z-Spread, interest rate vol (), and vol of vol ().
   * Compute convexity as .
   * Standardise and clean data to remove structural breaks and extreme outliers.
2. **Mean Reversion Parameter Estimation**:
   * Estimate parameters using historical data.
   * Perform Ordinary Least Squares (OLS) regression on OAS and Convexity.
   * Perform Maximum Likelihood Estimation (MLE) on OAS and Convexity, using OLS esimates as initial guess.
   * Conduct stationarity tests (e.g., Augmented Dickey-Fuller) to validate model assumptions.
   * Use the discretized equations to simulate the mean-reverting processes and compare with historical data for validation.
3. **Residual Variance Calibration**:
   * Fit the convexity-dependent variance function via non-linear least squares.
   * Assess statistical significance of convexity-driven uncertainty effects.
   * Incorporate the discretized variance function into the simulation model to capture the impact of convexity on OAS innovations.
4. **Global Model Calibration**:
   * Use numerical optimisation (e.g., maximum likelihood estimation, Nelder-Mead) to calibrate parameters.
   * Validate performance using out-of-sample tests.
   * Implement the discretized model equations in the calibration process to ensure consistency between the continuous-time model and its discrete-time implementation.
5. **Discretization of the System of SDEs**:
   * Discretize the continuous-time differential equations using the Euler-Maruyama method.
   * Define the time step size and the number of steps .
   * Initialize parameters and variables:
     + Set initial values for and .
     + Define the parameters .
   * Iterate over time steps:
     + For each time step :
       - Generate random variables and .
       - Update and using the discretized equations.
6. **Monte Carlo Simulation for Expected OAS**:
   * Run multiple simulations (e.g., 1000 paths) to estimate the expected value of OAS in one year.
   * For each simulation:
     + Simulate the OAS and Convexity processes using the discretized equations.
     + Collect the OAS estimate at the end of the simulation.
   * Calculate the expected value of OAS by averaging the results of all simulations.
7. **Sensitivity Analysis and Stress Testing**:
   * Simulate model responses to diverse economic scenarios.
   * Test extreme cases such as volatility spikes and rapid prepayment shifts.
8. **Operational Implementation**:
   * Automate parameter updates.
   * Incorporate real-time adjustments to account for shifts in rate vol and prepayment behaviour.

## 3. Alternative Approaches

| Methodology | Description | Strengths | Weaknesses | Optimal Use Case |
| --- | --- | --- | --- | --- |
| **Joint Reversion Model (Proposed)** | Mean-reverting dynamics for OAS and Convexity | Captures structural dependencies, incorporates exogenous factors | Requires careful calibration | Time-series valuation modelling |
| **Panel Regression** | Cross-sectional analysis of MBS spreads | Leverages large datasets | Ignores time-series dependencies, requires large cross-sectional sample | Index aggregated spread dynamics |
| **State-Space Model (SSM)** | Kalman filter-based latent factor modelling | Captures hidden states dynamically | Computationally demanding | High-frequency filtering applications |
| **Markov-Switching Model (MSM)** | Regime-switching framework | Models non-linear transitions | Hard to estimate transition probabilities | Capturing regime-dependent spread shifts |